Caluclus AB NAME: ______________________

MC Packet 3 - Derivatives and Tangent Lines

PERIOD: ______

In-Class Together: Problems 1-6

(1) An equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y=-7x+6$ (B) $y=-6x+5$ (C) $y=-2x+1$

(D) $y=2x-3$ (E) $y=7x-8$

(2) The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

(3) $\frac{d}{dx}\left(\ln e^{2x}\right)=$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

④ If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\dfrac{2x}{x^2-1}\right|$

(B) $\dfrac{2x}{\left|x^2-1\right|}$

(C) $\dfrac{2|x|}{x^2-1}$

(D) $\dfrac{2x}{x^2-1}$

(E) $\dfrac{1}{x^2-1}$

⑤ If $f(x) = \dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1+\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2}-1$ (E) $1-\dfrac{\pi}{2}$

⑥ If $y = \arctan(\cos x)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\dfrac{1}{(\arccos x)^2 + 1}$ (E) $\dfrac{1}{1+\cos^2 x}$

In-Class: Problems 7-16

(7) If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

(8) If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

(9) $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

(10) An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

(11) If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

(12) If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{-6x}{\left(4+x^2\right)^2}$ (B) $\dfrac{3x}{\left(4+x^2\right)^2}$ (C) $\dfrac{6x}{\left(4+x^2\right)^2}$ (D) $\dfrac{-3}{\left(4-x^2\right)^2}$ (E) $\dfrac{3}{2x}$

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(13) An equation of the line tangent to the graph of $y=\dfrac{2x-3}{3x-2}$ at the point $(1,5)$ is

(A) $13x-y=8$ (B) $13x+y=18$ (C) $x-13y=64$

(D) $x+13y=66$ (E) $-2x+3y=13$

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(14) $\dfrac{d}{dx}\cos^2(x^3)=$

(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

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(15) If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

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(16) The slope of the line tangent to the graph of $y=\ln\left(x^2\right)$ at $x=e^2$ is

(A) $\dfrac{1}{e^2}$ (B) $\dfrac{2}{e^2}$ (C) $\dfrac{4}{e^2}$ (D) $\dfrac{1}{e^4}$ (E) $\dfrac{4}{e^4}$

Homework: Problems 17-31

17. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

18. $\frac{d}{dx}\left(2^x\right) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2^x}{\ln 2}$

19. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

20. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$

(D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$

21. If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

(22) $\frac{d}{dx}\ln\left(\frac{1}{1-x}\right) =$

(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

(23) An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y-1 = -\left(x-\frac{\pi}{4}\right)$

(B) $y-1 = -2\left(x-\frac{\pi}{4}\right)$

(C) $y = 2\left(x-\frac{\pi}{4}\right)$

(D) $y = -\left(x-\frac{\pi}{4}\right)$

(E) $y = -2\left(x-\frac{\pi}{4}\right)$

(24) Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

(25) If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

(26) If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

(27) If f and g are twice differentiable and if $h(x) = f(g(x))$, then $h''(x) =$

(A) $f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

(B) $f''(g(x))g'(x) + f'(g(x))g''(x)$

(C) $f''(g(x))[g'(x)]^2$

(D) $f''(g(x))g''(x)$

(E) $f''(g(x))$

(28) What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

(29) If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

(30) If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n! e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n! e^x$

(31) $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6