Caluclus AB NAME: ______________________

MC Packet 2 - Limits, Continuity, and Theorems

PERIOD: ______

In-Class Together: Problems 1-6

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(1) If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

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(2) If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \ne 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

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(3) $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

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4. If f is a function such that $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist.

(B) f is not defined at $x = 2$.

(C) The derivative of f at $x = 2$ is 0.

(D) f is continuous at $x = 0$.

(E) $f(2) = 0$

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5. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.

(B) $f'(-1) = 0$

(C) The graph of f has a horizontal asymptote.

(D) The graph of f has a horizontal tangent line at $x = 3$.

(E) The graph of f intersects both axes.

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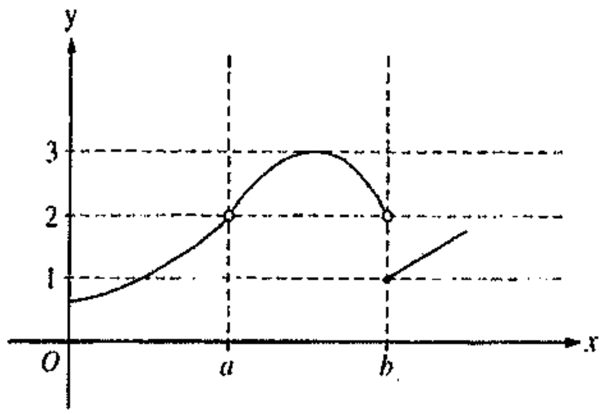
6. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x, which of the following must be true?

I. $f'(x) \leq g'(x)$ for all real x

II. $f''(x) \leq g''(x)$ for all real x

III. $\int_0^1 f(x)\,dx \leq \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III



7 The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 2$

(C) $\lim_{x\to b} f(x) = 2$

(D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

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8 If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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9 Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

(10) If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

(11) If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$

(B) $\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$

(C) $\lim_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$

(E) $\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$

(12) If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

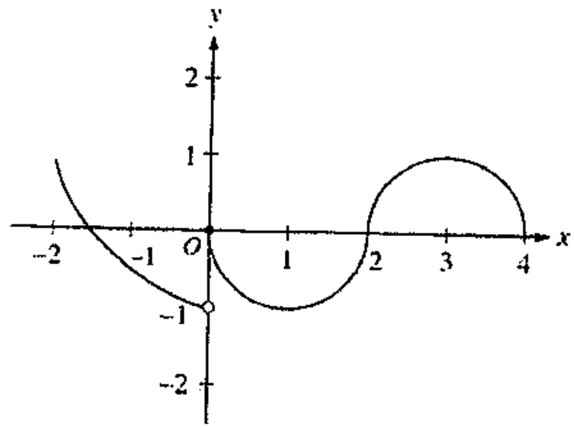
(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

(13) If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

(14) $\lim_{x\to 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞



(15) The graph of the function f shown in the figure above has a vertical tangent at the point $(2, 0)$ and horizontal tangents at the points $(1, -1)$ and $(3, 1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

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(16) At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

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(17) Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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(18) $\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

(19) If f is a continuous function on $[a,b]$, which of the following is necessarily true?

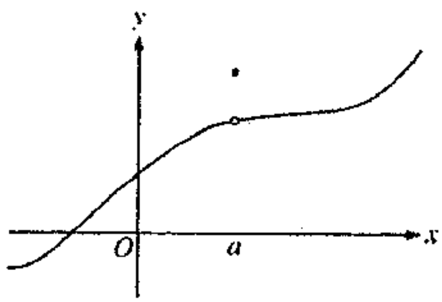
(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim\limits_{x\to x_0} f(x) = f\left(\lim\limits_{x\to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x)=0$ for some $x\in[a,b]$

(E) The graph of f' is a straight line.



(20) The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x=a$.

(B) f has a relative maximum at $x=a$.

(C) $x=a$ is in the domain of f.

(D) $\lim\limits_{x\to a^-} f(x)$ is equal to $\lim\limits_{x\to a^-} f(x)$.

(E) $\lim\limits_{x\to a} f(x)$ exists.

(21) The $\lim\limits_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

(22) $\lim\limits_{\theta\to 0}\dfrac{1-\cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) 1 (E) nonexistent

(23) Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0. \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x = 0$.

(C) f has a relative maximum.

(D) $f'(0) = 0$

(E) $f'(x) > 0$ for $x \neq 0$

(24) If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \to a} \frac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

(25) If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

(26) If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

(27) $\lim_{n \to \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent