Min/Max/Inflection FRQ 1
Mrs. Dicken

Let f be the function defined by $f(x) = kx^{1/2} - x^{3/2}$ for $x \geq 0$, where k is a constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k determine whether f has a relative minimum, relative maximum, or neither at $x = 1$.

(c) Does f have one critical number that is the same for every value of the constant k?

(d) For k a positive constant, on what interval(s) in terms of k will f be increasing and concave down?

(e) For k a positive constant, on what interval(s) in terms of k will f be decreasing and concave up?

(f) For a certain value of the constant k, the graph of f has a point of inflection when $f(x) = -1$. Find this value of k.

Solutions

(a) $f'(x) = \dfrac{k}{2}x^{-1/2} - \dfrac{3}{2}x^{1/2} = \dfrac{x^{-1/2}}{2}(k - 3x)$

$f''(x) = -\dfrac{k}{4}x^{-3/2} - \dfrac{3}{4}x^{-1/2} = -\dfrac{x^{-3/2}}{4}(k + 3x)$

(b) $k = 3$ and at $x = 1$ we have a relative maximum since $f''(1) < 0$.

(c) Yes, for any value of k, $f'(x)$ is undefined when $x = 0$.

(d) $\left(-\dfrac{k}{3}, \dfrac{k}{3}\right)$

(e) None

(f) $k = -3\left(\dfrac{1}{2}\right)^{2/3}$