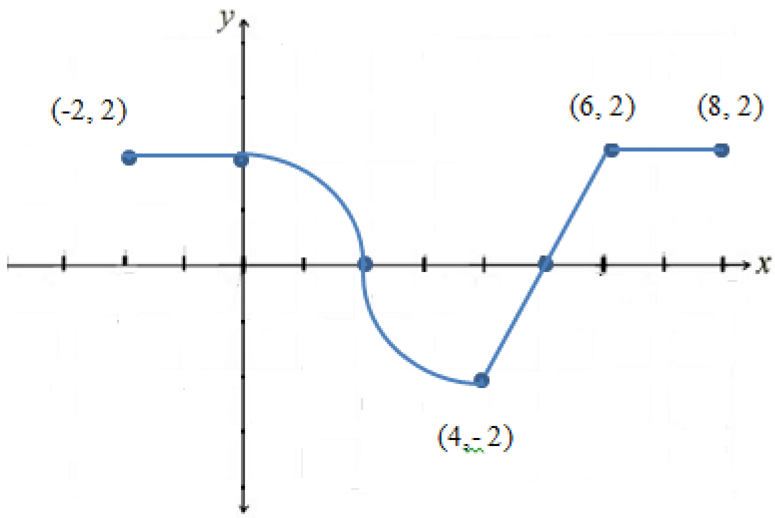
Jagged Line FRQ 2

Mrs. Dicken

The function f is defined on the closed interval $[-2, 8]$. The graph of f, given below, consists of three line segments and two quarter circles of radius 2. Let g be the function given by

$$g(x) = \int_x^4 f(t)\, dt$$



(a) Compute or state that it does not exsist:

$$g(8),\ g(2),\ g(0),\ g'(0),\ g'(4),\ g'(7),\ g''(-1),\ g''(4),\ g''(5)$$

(b) On what open interval(s) in $(-2, 8)$ is the graph of g both increasing and convcave up? Decreasing and concave up? Justify your answer.

(c) At what value(s) of x does g have a point of inflection? Justify your answer.

(d) Find the value(s) of x where $g(x) = 0$. Justify your answer.

(e) The function g is defined by $h(x) = g(3x^2 - 6)$. Find $h'(2)$.

(f) Let $k(x) = g(x) + x$ on $(-2, 8)$. Where are the critical numbers of k? Classify them as a local max, local min or neither. Justify your answer.

Solutions

Note: $g(x) = -\int_4^x f(t)\,dt$ so $g'(x) = -f(x)$ and $g''(x) = -f'(x)$.

(a) $g(8) = -4$
$g(2) = -\pi$
$g(0) = 0$
$g'(0) = -f(0) = -2$
$g'(4) = -f(4) = 2$
$g'(7) = -f(7) = -2$
$g''(-1) = -f'(-1) = 0$
$g''(4) = -f'(4) = DNE$
$g''(5) = -f'(5) = -2$

(b) g increasing and concave up when $g'(x) = -f(x)$ is positive and increasing, hence where $f(x)$ is negative and decreasing - $(2, 4)$. g is decreasing and concave up when $g'(x) = -f(x)$ is negative and increaseing, hence where $f(x)$ is positive and decreasing - $(0, 2)$.

(c) Where $g''(x) = -f'(x)$ changes sign, $x = 4$.

(d) $g(x) = 0$ when the area above equals area below the x-axis under $f(x)$ starting at $x = 4$, so $x = 4, 0, 6$.

(e) $h'(x) = g'(3x^2 - 6) \cdot 6x$ so $h'(2) = g'(6) \cdot 12 = -f(6) \cdot 12 = -24$

(f) Critical number of k where $k'(x) = g'(x) + 1 = -f(x) + 1$ is zero or undefined on $(-2, 8)$. That is where $f(x) = 1$ or where $f(x)$ is undefined. When $f(x) = 1$ since the radius of the circle is 2 we get that $x = \sqrt{3}$. Since $k''(x) = g''(x) = -f'(x)$ and $k''(\sqrt{3}) = -f'(\sqrt{3}) > 0$, $x = \sqrt{3}$ is a local min of k.