Jagged Line FRQ 1

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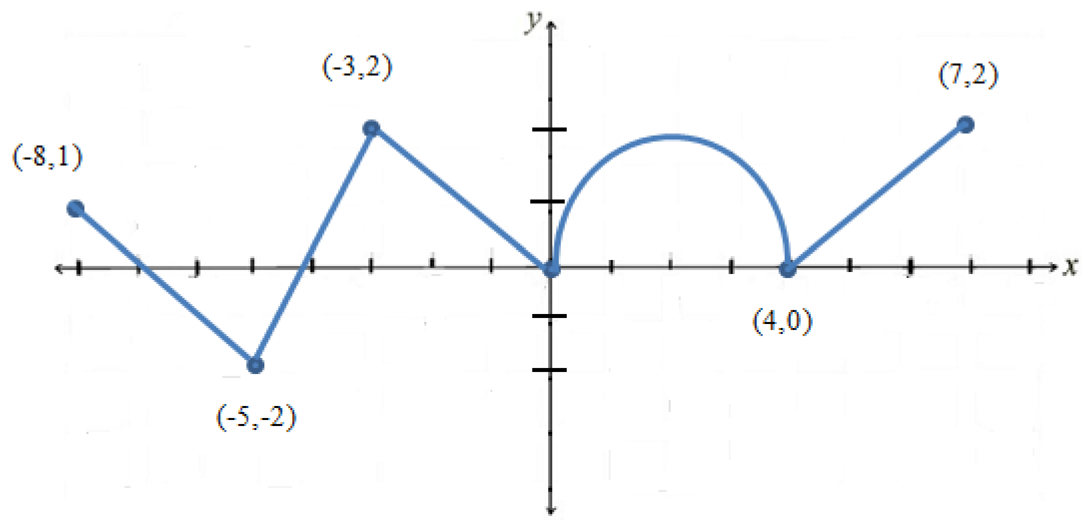
The function f is defined on the closed interval $[-8, 7]$. The graph of f, given below, consists of four line segments and one semicircle of radius 2. Let h be the function given by

$$h(x) = \int_{-3}^{x} f(t)\, dt$$



(a) Find $h(4)$ and $h'(2)$.

(b) On what open interval(s) in $(-8, 7)$ is the graph of h both increasing and convcave down? Justify your answer.

(c) At what value(s) of x does h have a point of inflection? Justify your answer.

(d) Find the value(s) of x where $h(x) = 0$. Justify your answer.

(e) The function g is defined by $g(x) = h(x)/3x^2$. Find $g'(2)$.

(f) The function p is defined by $p(x) = f(x^3 + x^2 - 6)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

Solutions

(a) $h(4) = 3 + 2\pi$ and $h'(2) = f(2) = 2$

(b) When $h'(x) = f(x) > 0$, h is increasing and when $h''(x) = f'(x) < 0$ or when $h'(x) = f(x)$ is decreasing then h is concave down. So on the intervals with f is both positive and decreaseing, $(-8,-7) \cup (-3,0) \cup (2,4)$

(c) When $h''(x) = f'(x)$ changes sign or when $h'(x) = f(x)$ changes from increasing to decreasing or vice versa, so at $x = -5, -3, 0, 2, 4$ h has inflection points.

(d) $h(x) = 0$ when the area above equals area below the x-axis under $f(x)$ starting at $x = -3$, so $x = -5, -3$.

(e) $g'(x) = \frac{h'(x)3x^2 - h(x)6x}{9x^4}$ thus as $h(2) = 3 + \pi$ and $h'(2) = f(2) = 2$ we have $g'(2) = \frac{-1-\pi}{12}$

(f) $p'(x) = f'(x^3 + x^2 - 6)(3x^2 - 2x)$ so $p'(-1) = f'(-6)(5) = -5$