Calculus AB: Review Sheet - Chapters 35-70

<u>Chapter 35</u>

Integration Rules

1. Constant Rule. If k is any real number, then the indefinite integral of k with respect to x is

$$\int k\,dx = kx + C$$

2. Coefficient Rule. Given any real number coefficient a and integrable function f,

$$\int af(x)\,dx = a\int f(x)\,dx$$

We've already seen:

- $\int ax^n\,dx = \dfrac{ax^{n+1}}{n+1} + c$

$n \neq -1$

<u>Chapter 36:</u> Critical Numbers

- A Critical number of a function f is a number c in the domain of f where either f'(c)=0 or f'(c) does not exist
- Derivative of f= 0 whenever the slope of the tangent line is drawn to the graph of f is horizontal
- Derivative does not exist when the graph of the function comes to a sharp point or where tangent line to the graph becomes vertical
- If f is a local max or min at c, then c is a critical number
- How to find a Critical Number:
  - Find the derivative of the function
  - Set the derivative equal to 0 or determine where the derivative is undefined
  - Factor and determine the critical number
  - Practice: find the critical number for the function f(x)= x^3 – 9/2x^2+6x+3
- Use Critical Numbers to find local max/min

<u>Chapter 37:</u> Differentiation by substitution (u-substitution)

- Before doing any u-substitution, you must know the basic integrals very well

$$\int dx = x$$

$$\int a\,dx = ax$$

$$\int a \bullet f(x)\,dx = a\int f(x)\,dx$$

$$\int (u+v)\,dx = \int u\,dx + \int v\,dx$$

$$\int u\,dv = u\int dv - \int v\,du = uv - \int v\,du$$

$$\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln(x)$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{;except n= -1}$$

The substitution Rule

If u = g(x) is a differentiable function and if f is continuous over the range of g(x), then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

The Substitution Rule for Definite Integrals

Example 1

$$\int_1^e \frac{\ln x}{x}\,dx$$

Example 2

$$\int_1^2 \frac{dx}{(3-5x)^2}$$

If u = g(x) is a differentiable function whose range is an interval I and f is continuous on I, AND g' is continuous on [a, b], then:

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

- Example:

$$\int x\sqrt{2x+1}\,dx,\qquad \text{let } u=2x+1,\ du=2dx,\ \therefore x=\frac{1}{2}(u-1) \text{ and } dx=\frac{1}{2}du$$

$$=\int \frac{1}{2}(u-1)u^{1/2}\,\frac{1}{2}du$$

Extract constants, distribute

$$=\frac{1}{4}\int(u^{3/2}-u^{1/2})du$$

Integrate (Power Rule)

$$=\frac{1}{4}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)+C$$

Re-substitute and simplify

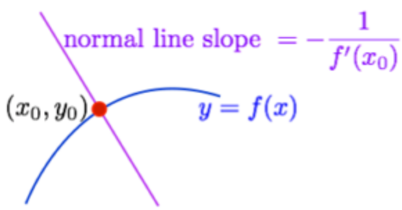
$$=\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}+C$$

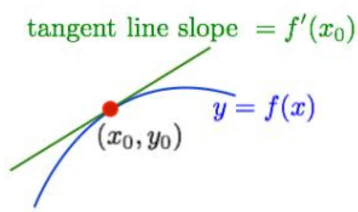
<u>Chapter 38:</u> Integral of a Sum and integral of 1/x

- $\int (f+g)\,dx=\int f\,dx+\int g\,dx$
- $\int x^{-1}\,dx=\int 1/x\,dx=\ln|x|+C$

<u>Chapter 40:</u> Normal lines, Units for the derivative, Max and Mins on graphing calculator

- The unit of the derivative of a function is the unit of the dependent variable (unit on the vertical axis) divided by the unit of the independent variable(unit on the horizontal axis)
- Normal lines:
  - Means perpendicular
  - The slope of the normal line is the negative reciprocal of the slope of the tangent line, b/c the normal line is perpendicular to the tangent line
  - Follow the same steps that you use for solving tangent line problems but change the slope by making it the negative reciprocal of the slope of the tangent line





<u>Chapter 41:</u> graphs of rational functions III; repeated factors

- For graphing rational functions:
  - Check that there are no holes
  - Determine the zeros
  - Determine the vertical asymptotes (in the denominator) and check if it stays the same or changes sign
  - Create a sign chart
- Repeated factors:
  - RF with even factor:
    - At zeros it will bounce
    - At vertical asymptotes, the sign does not change across the VA
  - RF with odd power
    - Same as power of 1

<u>Chapter 42:</u> Quotient Rule

- Low dee high minus high dee low, over the square of what's below: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

Chapter 44: Chain Rule; Alternate Definition of the derivative

- Chain Rule: Derivative of the outside, keep the inside, times the derivative of the inside

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Examples:

1. $\frac{d}{dx}(x^2+5)^8 = 8(x^2+5)^7 \cdot 2x = 16x(x^2+5)^7$
2. For $y = u^8$ and $u = x^2+5$,
$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (8u^7)(2x) = 8(x^2+5)^7 \cdot 2x = 16x(x^2+5)^7$

Alternate Definition of a Derivative

The slope of a secant line between x and x_0 is:

$$\frac{f(x)-f(x_0)}{x-x_0}$$

The derivative of a function provides us with a measure of the instantaneous rate of change. Thus, we get the derivative at x_0 or $f'(x_0)$ if we take the limit as the denominator goes to 0:

$$\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$$

Known as The Difference Quotient at a particular point

Chapter 45: Local Max and Min

- Step 1: Find the critical Numbers
- Step 2: 1st Derivative Test
  - Suppose f is continuous at a critical point $x0$.
  - If $f'(x)>0$ on an open interval extending left from $x0$ and $f'(x)<0$ on an open interval extending right from $x0$, then f has a relative maximum at $x0$.
  - If $f'(x)<0$ on an open interval extending left from $x0$ and $f'(x)>0$ on an open interval extending right from $x0$, then f has a relative minimum at $x0$.
  - If $f'(x)$ has the same sign on both an open interval extending left from $x0$ and an open interval extending right from $x0$, then f does not have a relative extremum at $x0$.
  - Relative extrema occur where f'(x) changes sign
- Step 2: 2nd Derivative Test
  - Suppose that c is a critical point at which $f'(c)=0$, that $f'(x)$ exists in a neighborhood of c, and that $f''(c)$ exists. Then f has a relative maximum value at c if $f''(c)<0$ and a relative minimum value at c if $f''(c)>0$. If $f''(c)=0$, the test is not informative.

Chapter 46: Related-Rates Problems

- Related Rates Problems:
  - Draw a picture, identify rates, label variables
  - Relate the variables
  - Differentiate to relate the rates
  - Solve for the missing rate

Chapter 47: Fundamental Theorem of Calculus, Part 1; Riemann Sums; The Definite Integral

Fundamental Theorem of Calculus

If a function f is continuous on the interval [a, b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any function such that $F'(x) = f(x)$ for all x in [a, b]

Riemann Sums

There are three types of Riemann Sums

Right Riemann:

$$A = \frac{b-a}{n}[f(x_1)+f(x_2)+f(x_3)\ldots f(x_n)]$$

Left Riemann:

$$A = \frac{b-a}{n}[f(x_0)+f(x_1)+f(x_2)+f(x_3)+\ldots f(x_{n-1})]$$

Midpoint Riemann:

$$A = \frac{b-a}{n}[f(x_{1/2})+f(x_{3/2})+f(x_{5/2})+f(x_{7/2})+\ldots f(x_{n-1/2})]$$

Chapter 48: Derivatives of Trigonometric Functions

$$\frac{d(\sin u)}{dx} = \cos u \frac{du}{dx}$$
$$\frac{d(\cos u)}{dx} = -\sin u \frac{du}{dx}$$
$$\frac{d(\tan u)}{dx} = \sec^2 u \frac{du}{dx}$$
$$\frac{d(\cot u)}{dx} = -\csc^2 u \frac{du}{dx}$$
$$\frac{d(\sec u)}{dx} = \sec u \tan u \frac{du}{dx}$$
$$\frac{d(\csc u)}{dx} = -\csc u \cot u \frac{du}{dx}$$

Chapter 49: Concavity and Inflection Points; First and Second derivative Tests

- 1st Derivative Test
  - Suppose f is continuous at a critical point $x0$.
  - If $f\prime(x) > 0$ on an open interval extending left from $x0$ and $f\prime(x) < 0$ on an open interval extending right from $x0$, then f has a relative maximum at $x0$.
  - If $f\prime(x) < 0$ on an open interval extending left from $x0$ and $f\prime(x) > 0$ on an open interval extending right from $x0$, then f has a relative minimum at $x0$.
  - If $f\prime(x)$ has the same sign on both an open interval extending left from $x0$ and an open interval extending right from $x0$, then f does not have a relative extremum at $x0$.
  - Relative extrema occur where f'(x) changes sign
- 2nd Derivative Test
  - Suppose that c is a critical point at which $f\prime(c)=0$, that $f\prime(x)$ exists in a neighborhood of c, and that $f\prime\prime(c)$ exists. Then f has a relative maximum value at c if $f\prime\prime(c) < 0$ and a relative minimum value at c if $f\prime\prime(c) > 0$. If $f\prime\prime(c)=0$, the test is not informative
- If the 2nd derivative is positive when x=c , the slope of the graph of function is increasing as the x-coordinate increases, and the graph is concave upward at that point
- If the 2nd derivative is negative when x=c, the slope of the graph of function is decreasing as the x-coordinate increases, and the graph is concave downward at that point
- The 1st derivative tells us whether the slope is positive, negative, or zero ad tells steep the slope if if the slope is not zero
- A zero value of the 2nd derivative does not necessarily indicate an inflection point. A zero value of the 2nd derivative compels us to use the first derivative test
- Point at which graph changes concavity: inflection point

Chapter 50: Derivatives of composite functions

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Examples:

1. $\frac{d}{dx}(x^2+5)^8 = 8(x^2+5)^7 \cdot 2x = 16x(x^2+5)^7$
2. For $y = u^8$ and $u = x^2 + 5$,
   $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (8u^7)(2x) = 8(x^2+5)^7 \cdot 2x = 16x(x^2+5)^7$

Chapter 51: Integration by Guessing

- Indefinite integral exists for every continuous function
- Integrate by guessing the answer then checking the guess by differentiating it; if the differential of the guess is the expression we r trying to integrate, it is the answer; if not , we guess again and check the new guess

Chapter 52: Absolute Max and Min Problems

- Step 1: Check if the function is continuous on a interval [a,b]
- Step 2: Find all the critical points of f(x) that are in that interval [a,b]
- Step 3: Find the end points

Chapter 54: Velocity and Acceleration; Motion due to gravity

- http://amazingbasisstudyguides.weebly.com/uploads/3/1/1/0/31109139/big_6_study_guide__edited_.pdf
- Motion due to gravity
  - Use up as the positive direction and down as the negative direction
  - Positive v(t) and a(t) r upward
  - A(t) due to gravity is negative since the objects released from above the surface of earth accelerate downward

Chapter 57: Properties of Definite Integral

I) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

II) $\int_b^b f(x)\,dx = 0$

III) *a)* $\int_a^b c\,dx = c(b-a)$ where c is a constant

b) $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

c) $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$ where c is a constant

IV) $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

V) If $f(x) \geq 0$ for $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq 0$

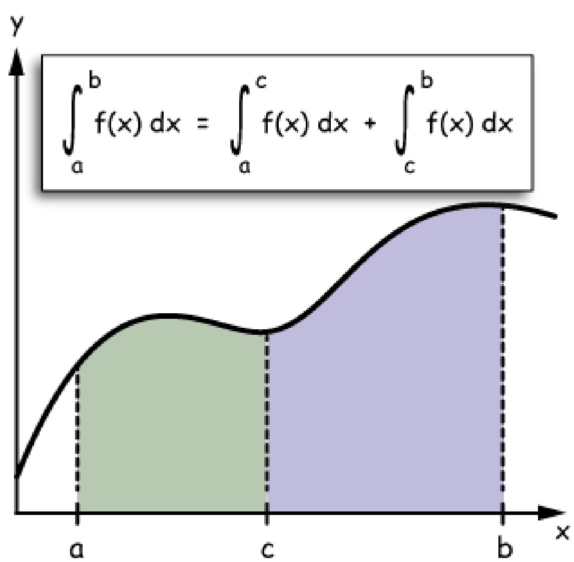
VI) If $f(x) \geq g(x)$ for $x \in [a,b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

VII) If $m \leq f(x) \leq M$ for $x \in [a,b]$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

Chapter 59: Computing Area; More Numerical Integration on a graphing calculator

- Total area:



Chapter 60: area between two curves

- $$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} dx, \qquad a \le x \le b$$
- $$A = \int_c^d \begin{pmatrix} \text{right} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{left} \\ \text{function} \end{pmatrix} dy, \qquad c \le y \le d$$
- Fundamental Theoram of Calculus:
  - $A = \int_a^b f(x) - g(x) dx$

Chapter 61: Playing games with f, f' and f''

- Remember this table

○ f	○ ↑/↓	○ Concave up/down
○ f'	○ +/-	○ ↑/↓
○ f''	○	○ +/-

Chapter 62: Work, Distance, Rates

- Work=force X distance traveled
- Units: joules

Chapter 63: Critical Number (Closed Interval) Theorem

- Extreme Value Theorem: If f is continuous on the closed interval t=[a,b], then f attains a maximum value M and a minimum value m on I
- Critical Number Theorem: If f is continuous function on a closed interval I and if f attains a max or min value at x=c, where c is in I, then either
  - 1. C is an endpoint
  - f'(c) does not exist, or
  - f'(c)=0

Chapter 64: Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\left[\sin^{-1}\left(\frac{x}{a}\right)\right] = \frac{1}{\sqrt{a^2 - x^2}}$$
$$\frac{d}{dx}\left[\cos^{-1}\left(\frac{x}{a}\right)\right] = -\frac{1}{\sqrt{a^2 - x^2}}$$
$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{a^2 + x^2}$$
$$\frac{d}{dx}\left[\csc^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{x\sqrt{x^2 - a^2}}$$
$$\frac{d}{dx}\left[\sec^{-1}\left(\frac{x}{a}\right)\right] = \frac{a}{x\sqrt{x^2 - a^2}}$$
$$\frac{d}{dx}\left[\cot^{-1}\left(\frac{x}{a}\right)\right] = -\frac{a}{a^2 + x^2}$$

Chapter 66: u-substitution; Change of variable

$$\int dx = x$$
$$\int a\,dx = ax$$
$$\int a \bullet f(x)dx = a\int f(x)dx$$
$$\int (u+v)dx = \int u\,dx + \int v\,dx$$
$$\int u\,dv = u\int dv - \int v\,du = uv - \int v\,du$$
$$\int \frac{1}{x}dx = \int x^{-1}\,dx = \ln(x)$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad \text{;except n= -1}$$

The substitution Rule

If u = g(x) is a differentiable function and if f is continuous over the range of g(x), then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

:

The Substitution Rule for Definite Integrals

Example 1 $\int_1^e \frac{\ln x}{x}\,dx$

Example 2 $\int_1^2 \frac{dx}{(3-5x)^2}$

If u = g(x) is a differentiable function whose range is an interval I and f is continuous on I, AND g' is continuous on [a, b], then:

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Chapter 67: Areas Involving Functions of y

- $\int (right\ f(y) - left\ f(y))dy$ y=b y=a

Chapter 68: Even and Odd Functions

- If a function is an odd function, then the same things happen to the graph of the function at equal distances to the left and right of the origin
  - f(-x)=-f(x)
- For any even function f and any value of x in its domain,
  - f(-x)=f(x)
- Even Functions:
  - For an even function the same things happen to the graph of an even function at equal distances to the left and the right of the origin
- A polynomial function is even if every exponent of x is even and odd if every exponent of x is odd (note a constant is x^0 so it's even)

Chapter 70: Properties of Limits; Some special limits

- The limit of a function f as x approaches c is a number, the number that f(x) approaches as x gets closer and closer to c

THEOREM 1.2 PROPERTIES OF LIMITS

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = K$$

1. Scalar multiple: $\lim_{x\to c} [b f(x)] = bL$
2. Sum or difference: $\lim_{x\to c} [f(x) \pm g(x)] = L \pm K$
3. Product: $\lim_{x\to c} [f(x)g(x)] = LK$
4. Quotient: $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
5. Power: $\lim_{x\to c} [f(x)]^n = L^n$