

**Continuity:**  $f(a)$  must exist.  $\lim_{x \rightarrow a} f(x) = f(a)$

### Limits

$\lim_{x \rightarrow a^+} f(x)$  must equal  $\lim_{x \rightarrow a^-} f(x)$  and both limits exist or else the limit DNE

$f(x)$	increasing / decreasing	positive / negative
$f'(x)$	positive / negative	increasing / decreasing
$f''(x)$		positive / negative

### Interval Notation

Closed Circles mean use brackets :  $[a, b]$

Open Circles mean use parentheses :  $(a, b)$

If you have an open then closed circle use a parenthesis then a bracket :  $(a, b]$

If you have two intervals  $a < x < b$  and  $c < x < d$  they can be in union so now write it as  $(a, b) \cup (c, d)$

### Properties of Log and Ln

$$\log_b mn = \log_b m + \log_b n$$

$$\log_b m - \log_b n = \log_b \frac{m}{n}$$

$$\log_b m^n = n \log_b m$$

$$\log_b m = \log_b n \text{ then } m=n$$

**Properties of Limits:** <http://bovcalculus.weebly.com/calculus-ab/lesson-17>

### Tangent Lines and Derivatives:

The slope of a curve at  $x=a$  is the slope of the tangent line at  $x=a$

$$\text{Derivative: } f'(x) \text{ or } \frac{d}{dx} f(x) = \lim_{x \rightarrow h} \frac{f(x+h) - f(x)}{h}$$

### Function Composition

$$(f+g)(x) = f(x) + g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$(f \circ g)(x) = f(g(x))$$

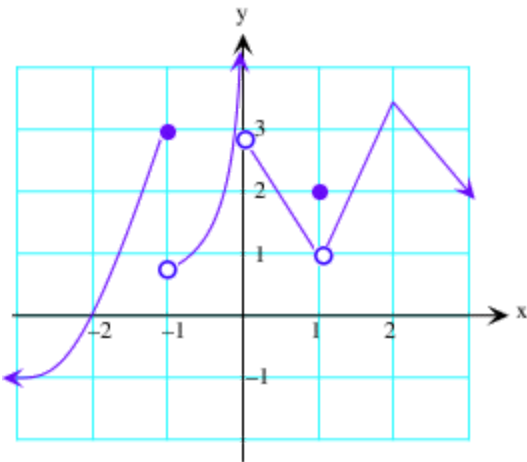
$$(g \circ f)(x) = g(f(x))$$

### Power Rule

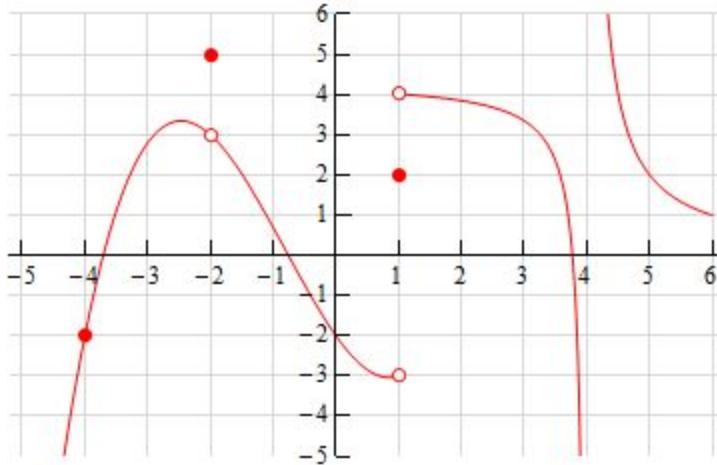
$$f(x) = x^n \quad f'(x) = n x^{n-1}$$

# Practice Problems

- 1.)  $f(x) = x(x+2)(x-3)$ 
  - a.) What interval(s) is  $f'(x) < 0$  ?
  - b.) What interval(s) is  $f'(x) > 0$  ?
  - c.) Where is  $f'(x) = 0$  ?
  - d.) Find all roots for  $f(x)$
- 2.)  $h(x) = -x(x-5)(x+1)$ , set  $y_{\max} = 50$  and  $y_{\min} = -50$ 
  - a.) What interval(s) is  $f'(x) < 0$  ?
  - b.) What interval(s) is  $f'(x) > 0$  ?
  - c.) Where is  $f'(x) = 0$  ?
  - d.) Find all roots for  $f(x)$



- 3.)
  - a.)  $\lim_{x \rightarrow 2^-} f(x) =$
  - b.)  $\lim_{x \rightarrow 1^-} f(x) =$
  - c.)  $\lim_{x \rightarrow -2^+} f(x) =$
  - d.)  $\lim_{x \rightarrow -1} f(x) =$
  - e.) Is  $f(x)$  continuous at  $x=1$ ? Justify your answer using the definition of continuity



- 4.)
- $\lim_{x \rightarrow 4} f(x) =$
  - $\lim_{x \rightarrow -2} f(x) =$
  - $\lim_{x \rightarrow 1^+} f(x) =$
  - $\lim_{x \rightarrow 1^-} f(x) =$
  - Is  $f(x)$  continuous at  $x=1$ ? Justify your answer using the definition of continuity
- 5.)  $\lim_{x \rightarrow \infty} \frac{x^2+4x+3}{x^4+5x+1} =$
- 6.)  $\lim_{x \rightarrow \infty} \frac{x^3-5x^4+7}{3x^3-9x+1} =$
- 7.)  $\lim_{x \rightarrow \infty} \frac{4x^4+5x^5+6}{x^4+2x+1} =$
- 8.) Use the definition of a derivative to find the derivative of each function
- $g(x) = x^3 + 1$
  - $f(x) = x^2 + 4x$
  - $h(x) = x^4 + x^2$
- 9.) Find the derivative of the following
- $A(x) = 6x^{-3} + 5x + \frac{1}{x^3}$
  - $B(x) = \sqrt[3]{x} + 4x^{-2}$
  - $V(x) = \frac{1}{\sqrt{x}} + 3x^{-5}$
- 10.) Solve for  $x$  in the following
- $\log_4(25x + 5) - \log_4 5 = 2$
  - $\log_8(4x + 3) + \log_8 7 = 2$
- 11.) Find the equation of the line tangent to  $f(x) = 4x^2 + 2x + 1$  at the point  $x=3$
- 12.) Find the equation of the line tangent to  $h(x) = 5x^3 + 4x^2$  at the point  $x=2$

# ANSWER KEY

1.)

- a.)  $(-1.120, 1.786)$
- b.)  $(-\infty, -1.120), (1.786, \infty)$
- c.)  $(1.786, -8.209), (-1.120, 4.061)$
- d.)  $x = -2, x = 0, x = 3$

2.)

- a.)  $(-\infty, -523), (3.189, \infty)$
- b.)  $(-523, 3.189)$
- c.)  $(3.189, 24.193), (-523, -1.378)$
- d.)  $x = -1, x = 0, x = 5$

3.)

- a.) 3
- b.) DNE
- c.)  $-\infty$
- d.) DNE
- e.) No, because  $\lim_{x \rightarrow 1} f(x) \neq f(1)$

4.)

- a.) DNE
- b.) DNE
- c.) 4
- d.) -3
- e.)  $\lim_{x \rightarrow 1^+} f(x) \neq \lim_{x \rightarrow 1^-} f(x) \neq f(1)$

5.) 0

6.)  $-\infty$

7.)  $\infty$

8.)

- a.)  $3x^2$
- b.)  $2x+4$
- c.)  $4x^3+2x$

9.)

- a.)  $\frac{-21}{x^4} + 5$
- b.)  $\frac{1}{3}x^{-2/3} - 8x^{-3}$
- c.)  $\frac{-1}{2}x^{-1} - 15x^{-6}$

10.)

- a.)  $X = \frac{17}{5}$
- b.)  $X = \frac{43}{25}$

11.)  $y = 26x - 35$

12.)  $y = 76x - 96$

