AP Calculus AB – "Big 6" Study Guide

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1. CRITICAL NUMBERS

a. First Derivative Test
   i. Local maximum at x = c if f' changes from (+) to (-)
   ii. Local minimum at x = c if f' changes from (-) to (+)
   iii. Inflection point when there is no change in the sign of f'

b. Second Derivative Test
   i. Local maximum at x = c if f is concave down at x = c, if f''(c) < 0
   ii. Local minimum at x = c if f is concave up at x = c, if f''(c) > 0
   iii. Inflection point where there is a change in concavity, where f''(x) changes sign

c. If f(x) concave down, then tangent line is above graph of f (tangent line an overestimate)
   If f(x) concave up, then tangent line is below graph of f (tangent line an underestimate)

d. Horizontal tangent line, where f'(x) = 0, where f'(x) crosses x-axis
   i. Critical numbers: x-value where f'(x) equals zero or is undefined (in domain of f)

e. f'(x) = Slope of the tangent line to f at x

f. Absolute minimum on closed interval: plug all critical numbers, including endpoints, into the function and determine which x-value gives you the smallest value of the function
   i. Shortcut – plug endpoints and local minima into function

g. Absolute maximum on closed interval: plug all critical numbers, including endpoints, into the function and determine which x-value gives you the largest function value
   i. Shortcut – plug endpoints and local maxima into function

h. Extreme Value Theorem:
   If f is a continuous function on [a,b] then f attains a maximum and minimum value on [a,b]

i. Critical Numbers Theorem:
   If f is a continuous function on [a,b] and f attains a maximum or minimum at x=c in [a,b] then either
   1. f'(c) = 0
   2. f'(c) is undefined
   3. c is an endpoint of [a,b]

2. INTEGRATION APPLICATIONS

a. If given a rate and you want items, then take the antiderivative
   i. Antiderivative of Rate (units/time) equals units

b. Meaning and interpretation of $\int_a^b f'(x)dx$
   i. This is the change in f(x) in units of f(x) over the interval a to b in units of x
   ii. Add areas above x-axis and subtract areas below x-axis

c. Fundamental Theorem of Calculus (FTC):
   i. if f is a continuous function on [a,b] and F, an antiderivative of f then

$$\int_a^b f(x)dx = F(b) - F(a)$$

   ii. Any continuous function has an antiderivative
   iii. F(x) = $\int_c^x f(t)dt$ because $\frac{d}{dx}$(F(x)) = f(x) and

$$\frac{d}{dx}\left(\int_c^x f(t)dt\right) = \frac{d}{dx}(F(x) - F(c)) = f(x)$$

d. Average value of a function f on [a,b],

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx \quad \text{(units of f)}$$

e. Average rate of change of f(x) on [a,b], $\frac{f(b)-f(a)}{b-a}$ (units of f / units of x)

f. Instantaneous rate of change of f(x) at x=c is f'(c)

g. Mean Value Theorem (MVT):
   If f is a continuous function on [a,b] and f is differentiable on (a,b) then there is at least one value c, a <c < b with:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

h. Intermediate Value Theorem:
   If f is continuous on [a,b] and Y is a number between f(a) and f(b) then there is at least one value c between a and b with f(c) = Y

i. UNITS! Check units and write units!

3. VOLUMES AND AREAS

a. Volume of a solid of revolution with disks:

$$\int_{x=a}^{x=b} \pi[f(x)]^2 dx \quad or \quad \int_{y=a}^{y=b} \pi[f(y)]^2 dy$$

b. Area of a region:

$$\int_{x=a}^{x=b} (top\ f(x) - bottom\ f(x))dx \quad \text{or} \quad \int_{y=a}^{y=b} (right\ f(y) - left\ f(y))dy$$

c. Volume of solid of revolution with washers:

$$\int_{x=a}^{x=b} (\pi R^2 - \pi r^2)dx \quad or \quad \int_{y=a}^{y=b} (\pi R^2 - \pi r^2)dy$$

$$About\ y = c: \quad V = \pi \int_{x=a}^{x=b} ((farthest\ f(x) - c)^2 - (closest\ f(x) - c)^2)dx$$

$$About\ x = c: \quad V = \pi \int_{y=a}^{y=b} ((farthest\ f(y) - c)^2 - (closest\ f(y) - c)^2)dy$$

i. dx if revolving around x-axis or axis parallel to the x-axis, y = c
ii. dy if revolving around y-axis or axis parallel to the y-axis, x = c

d. Volume of Cross-Sections:

$$\int_{x=a}^{x=b} A(x)dx \quad or \quad \int_{y=a}^{y=b} A(y)dy$$

i. With A(x) or A(y) as the cross-sectional area
ii. If cross sections are perpendicular t x-axis, then areas will be functions of x: A(x)
iii. If cross sections are perpendicular t y-axis, then areas will be functions of y: A(y)
iv. Cross-sectional area with base b, distance between the curves, is f – g

1. Square cross sections: $\int_a^b (b^2)dx$
2. Semicircle Cross sections (diameter base): $\int_a^b \left(\frac{\pi}{2}\right)\left(\frac{b}{2}\right)^2 dx$
3. Rectangle cross sections: $\int_a^b (bh)dx$
4. Equilateral triangle cross sections: $\int_a^b \left(\frac{\sqrt{3}}{4}\right)(b^2)dx$
5. Right Isosceles triangle cross sections:
   a. Hypotenuse as base: $\int_a^b \left(\frac{1}{4}\right)(b^2)dx$
   b. Leg as base: $\int_a^b \left(\frac{1}{2}\right)(b^2)dx$

4. RIEMANN SUMS

a. Area of Trapezoid:

$$\frac{1}{2}w(s_1 + s_2) = \frac{1}{2}\,(x_2 - x_1)(f(x_1) + f(x_2))$$

i. Width of Trapezoid: $w = x_2 - x_1$

ii. Sides of the Trapezoid: $s_1 = f(x_1)$ and $s_2 = f(x_2)$

b. Riemann sum with rectangles: sum areas of rectangles on specified intervals

c. Split your interval into sub-intervals,

i. Right Riemann sum:

1. Width of rectangle is width of sub-interval, Δx
2. Height of rectangle, $f(x)$ where x is furthest **right** on sub-interval

ii. Left Riemann sum:

1. Width of rectangle is width of sub-interval, Δx
2. Height of rectangle, $f(x)$ where x is furthest **left** on sub-interval

iii. Midpoint Riemann sum:

1. Width of rectangle is width of sub-interval, Δx
2. Height of rectangle, $f(x)$ where x is the **middle** of sub-interval

d. Under of Over Estimates for $\int_a^b f(x)dx$

i. Right Riemann sum:

1. Over Estimate if f(x) is increasing
2. Under Estimate if f(x) is decreasing

ii. Left Riemann sum:

1. Under Estimate if f(x) is increasing
2. Over Estimate if f(x) is decreasing

e. Meaning and interpretation of $\int_a^b f'(x)dx$

i. This is the change in f(x) in units of f(x) over the interval a to b in units of x

ii. Add areas above x-axis and subtract areas below x-axis

5. VELOCITY AND ACCELERATION

a. Position: $x(t) = x(0) + \int_0^t v(s)ds$

b. Velocity: v(t) = x`(t)
$v(t) = v(0) + \int_0^t a(s)ds$

c. Acceleration : a(t) = v`(t) = x``(t)

d. Speed: $|v(t)|$
   i. If v(t) and a(t) have same sign then speed is increasing
   ii. If v(t) and a(t) have different signs then speed is decreasing

e. $\int_a^b v(t)dt$ = Displacement/Change in Position on the interval [a,b]

f. $\int_a^b |v(t)|dt$ = Total Distance Traveled on the interval [a,b]

g. Average velocity on [a,b] = Average rate of change of position on [a,b]

$$\frac{1}{b-a}\int_a^b v(t)dt = \frac{x(b)-x(a)}{b-a}$$

h. Average acceleration on [a,b] = Average rate of change of velocity on [a,b]

$$\frac{1}{b-a}\int_a^b a(t)dt = \frac{v(b)-v(a)}{b-a}$$

i. Particle farthest left → compare positions at endpoints and local minima to find absolute minimum (Don't forget initial position)
   i. "Find time at which particle is farthest to left" = time t where x(t) is absolute minimum

j. Particle farthest right → compare positions at endpoints and local maxima to find absolute maximum (Don't forget initial position)
   i. "Find time at which particle is farthest to right" = time t where x(t) is absolute maximum

k. Particle at rest →when v(t) = 0

l. Particle changing direction → when v(t) changes sign

m. Use IVT when it wants a specific y-value, MVT for the derivative of a function (Cannot apply MVT on intervals where function has corner or vertical tangent line)

n. v(t)< 0 means particle is moving left, v(t)>0 means particle is moving right

o. x(t)<0 means particle is left of origin, x(t)>0 means particle is right of origin

6. DIFFERENTIAL EQUATIONS

a. Make a table to find the slopes for the slope-field graph
   i. Shortcut: horizontal line where f'(x)=0
   ii. 0 in denominator means the function is undefined at those points (write nothing on the graph for that point)

b. When finding domain of the solution to a differential equation:
   i. Need one continuous interval where the solution is defined and differentiable and the interval must include the initial condition
   ii. There can never be a negative number inside a square root
   iii. There can never be zero on the denominator
   iv. When dividing both sides of the inequality by a negative, the inequality flips
      1. Ex) $-x > -4 \rightarrow x<4$

c. Remember the **absolute value** when integrating $\frac{1}{y}$ or $\frac{1}{x}$

$$\int \frac{1}{x}dx = \ln|x| + C \qquad and \qquad \int \frac{1}{1-x}dx = -\ln|1-x| + C$$

   i. Keep the absolute value until you determine if the inside of the absolute value is (+) or (-) based on the **initial condition**
   ii. If initial condition makes absolute value negative then negate one side of the equation

d. Don't forget the **+C** !!!!!!!!

e. Remember that taking the square root of both sides gives a positive and negative answer.
   i. Sign of answer is determined by the sign of the y-value of the initial condition

f. Cosine is an even function, so cos(-x) = cos(x)
g. Sine is an odd function, so sin(-x) = - sin(x)