Sample Homework

YOUR NAME HERE
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Math Review

1. First to use the math environment to insert and equation or formula use the single or double dollar signs about the expression like so: $3x + 5x^2 = 8$ or

$$3x + 5x^2 = 8.$$

To create an equation on multiple lines use the 'align' environment:

$$\begin{aligned}
4(x+3) - 3(-x-2) &= 4x + 12 - 3(-x) - 3(-2) \quad \text{[By Distributive Law]} \\
&= 4x + 12 + 3x + 6 \quad \text{[By Associative Law]} \\
&= 7x + 18 \quad \text{[By commutative Law]}
\end{aligned}$$

2. Here are some basic math commands in the .tex file and their outputs in the .pdf file:

For fractions:

$$\frac{1}{2} \cdot \frac{1 + x + x^2}{x^3 + x + 4}$$

For summation and product notation:

$$\sum_{i=0}^{n+1} (i^2 + 2) \quad \text{and} \quad \prod_{i=0}^{n-1} i + 3 \quad \text{and} \quad \left(\sum_{i=-n}^{n} i^2 \right)$$

For square roots: $\sqrt{2}$, $\sqrt{10 - x^2}$, n^{th} roots: $\sqrt[n]{1 + x + x^2}$

For logarithms and exponents: $\log_3(x^4)$, $4^{\log_4 x}$

Inequalities: $x \geq 0$ and $x \leq 0$

To create a list of numbers: $\ldots,\ -1,\ 0,\ 1,\ 2,\ 3,\ \ldots$

Sets of numbers:

$$\mathbb{R}, \quad \mathbb{Z}^+, \quad \mathbb{Q}_{\geq 0}, \quad \{0,\ 1,\ 2,\ 3\}, \quad \{x \in \mathbb{Z} \mid x \geq 0\} = \mathbb{Z}_{\geq 0}$$

Basic set operators: $A \cap B$, $\quad A \cup B$, $\quad A - B$, $\quad \overline{A} = U - A$, $\quad A \subseteq B$, $\quad A \supset B$

Power set: $\mathcal{P}(\emptyset) = \{\emptyset,\ \{\emptyset\}\}$

Boxing your solutions:

$$\boxed{x^2 + y^2 = z^2}$$

Section 1.1 - Logic

Some logic symbols:

- AND: $\wedge$
- OR (inclusive): $\vee$
- XOR (exclusive): $\oplus$
- Negation: $\neg$
- Equivalence: $\equiv$
- Implication: $\leftarrow$, $\rightarrow$, $\Rightarrow$
- Biimplication: $\leftrightarrow$ or $\Leftrightarrow$
- True and False: **T** and **F**
- Universal Quantification: $\forall$
- Existential Quantification: $\exists$
- Therefore: $\therefore$

Section 1.7 - Proofs

1. Prove that if $x \in \mathbb{Z}$ is even, then $x^2 + 1$ is odd.

   Proof. (Direct):

   Let $x \in \mathbb{Z}$ and suppose that x is even. Then by definition $x = 2k$ for some $k \in \mathbb{Z}$. Next consider $x^2 + 1$.

   $$\begin{aligned} x^2 + 1 &= (2k)^2 + 1 \\ &= 4k^2 + 1 \\ &= 2(2k^2) + 1 \end{aligned}$$

   Since $n = 2k^2 \in \mathbb{Z}$ we have that $x^2 + 1 = 2n + 1$ for $n \in \mathbb{Z}$ and hence by definition $x^2 + 1$ is add. $\square$

2. Prove or disprove: the subtraction of two irrational numbers is irrational.

   Disproof. (Counterexample):

   Consider $\sqrt{2}$ which is irrational. However

   $$\sqrt{2} - \sqrt{2} = 0$$

   and as $0 \in \mathbb{Q}$ this contradicts the statement above, hence it is false.

Section 2.3 - Functions

Floor function: $\lfloor x \rfloor$

Ceiling function: $\lceil x \rceil$

Functions composition: $f \circ g : \mathbb{Z} \to \mathbb{Z}$

Piecewise Functions:

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ -\frac{n+1}{2} & \text{if } n \text{ is odd} \end{cases}$$

Graphing:

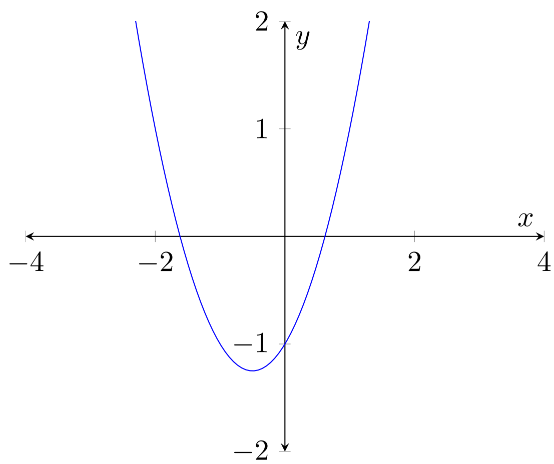
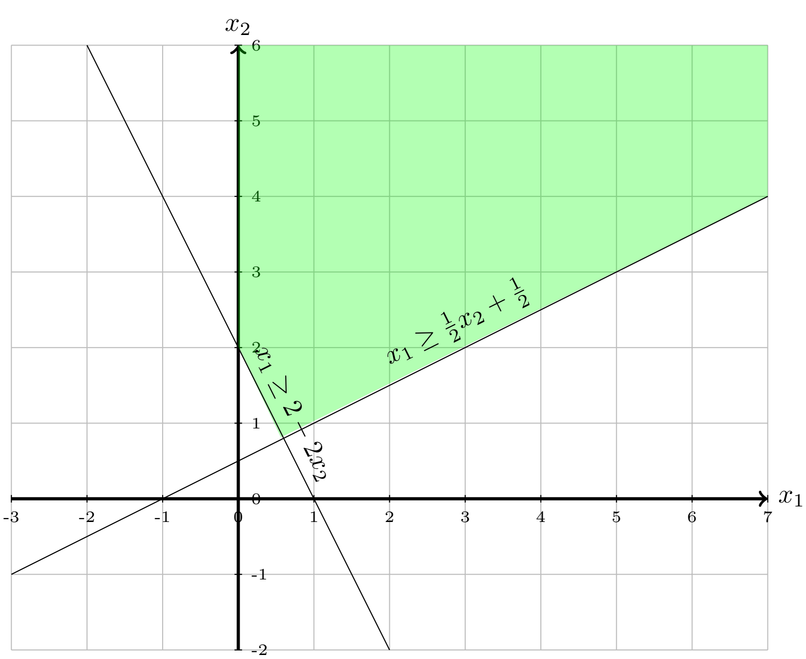


Figure : Graph of $f(x) = x^2 + x - 1$



Section 2.6 - Matrices

For each part of this problem below, consider the matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 & -1 \\ -5 & 7 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Compute $A + C$.

$$A + C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}$$

(b) Compute $A \cdot B$.

$$A \cdot B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 & -1 \\ -5 & 7 & 0 \end{bmatrix} = \begin{bmatrix} -7 & 18 & -1 \\ -5 & 7 & 0 \end{bmatrix}$$

(c) Compute C^2.

$$C^2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(d) Compute $I_3 \odot I_3 \odot I_3$.

$$I_3 \odot I_3 \odot I_3 = I_3 \odot I_3 = I_3$$

A general $m \times n$ matrix:

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

Section 3.1 - Algorithms

- To write code in LaTeX that looks like actual code use verbatim:

```
for i in range(1, 5):
  print i
else:
  print "The for loop is over"
```

- To write an algorithm in pseudo code use the algorithm 2e package, which is already in the preamble,

```
Data: this text
Result: how to write algorithm with LaTeX2e
initialization;
while not at end of this document do
    read current;
    if understand then
        go to next section;
        current section becomes this one;
    else
        go back to the beginning of current section;
    end
end
```

Algorithm 1: How to write algorithms

- Here is another example:

```
PrintAll( node v ):
    for each (item in v as x in order) {
        if x == is a key
            print x
        else PrintAll(x)
    };
```

Section 6.4 - Counting

Permutations:

$$P(n, r) = \frac{n!}{(n-r)!} = \binom{n}{r} \cdot r!$$

Combinations:

$$C(n, r) = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

Section 9.1 - Relations

Comparable: $\succeq$, $\preceq$

Here are some graph representations of relations, the first one is the kind we'll use in this course whereas the second you may see in Discrete Structures or Algorithms. I just included it to give you more options.

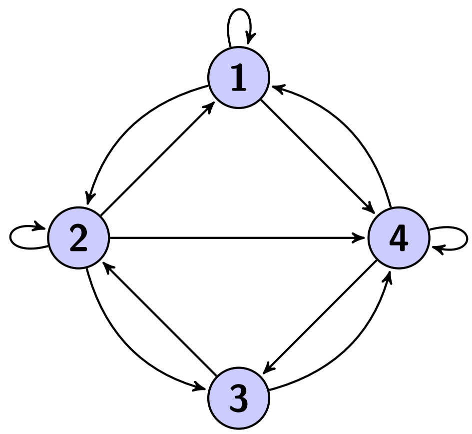


Figure 1: Relation of the set $\{1,\ 2,\ 3,\ 4\}$

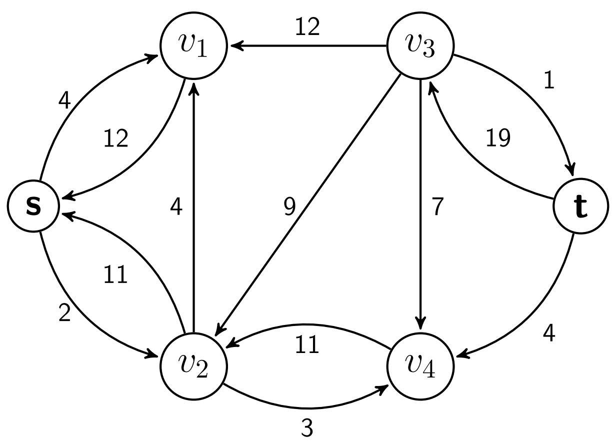


Figure 2: Network Flow

Greek Letters

- alpha: A, α
- beta: B, β
- gamma: Γ, γ
- delta: Δ, δ
- epsilon: E, ϵ
- theta: Θ, θ
- Lambda: Λ, λ
- mu: M, μ
- sigma: Σ, σ
- phi: Φ, ϕ
- psi: Ψ, ψ
- omega: Ω, ω

Calculus

1. Limits:

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \to 0^-} \frac{\sin x}{x} = 1 \quad \text{thus} \quad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

2. Derivative Notation:

$$y' = f'(x) = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx} f(x)$$

$$y'' = f''(x) = \frac{d^2 f}{dx^2} = \frac{d^2 y}{dx^2} = \frac{d^2}{dx^2} f(x)$$

3. Integrals:

$$\int \cos x \ dx = \sin x + C$$

$$\int_0^2 x \ dx = 2$$

4. Symbols:
   - Partial: ∂
   - Delta: Δ